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Optimisation Algorithms for the Charge  
Dispatch of Plug-in Vehicles Based on  
Variable Tariffs

## **Abstract**

Plug-in vehicles powered by renewable energies are a viable way to reduce local and total emissions and could also support a highly efficient grid operation. Indirect control by variable tariffs is one option to link charging or even discharging time with the grid load and the renewable energy production. Algorithms are required to develop tariffs and evaluate grid impacts of variable tariffs for electric vehicles (BEV) as well as to schedule the charging process optimisation. Therefore a combinatorial optimisation algorithm is developed and an algorithm based on graph search is used and customised. Both algorithms are explained and compared by performance and adequate applications. The developing approach and the correctness of the quick combinatorial algorithm are proved within this paper. For vehicle to grid (V2G) concepts, battery degradation costs have to be considered. Therefore, common life cycle assumptions based on the battery state of charge (SoC) have been used to include degradation costs for different Li-Ion batteries into the graph search algorithm. An application of these optimisation algorithms, like the onboard dispatcher, which is used in the German fleet test "Flottenversuch Elektromobilität". Grid impact calculations based on the optimisation algorithm are shown.

## **Keywords**

BEV, V2G, Plug-In-Vehicles (PHEV), optimisation, mobile dispatcher, demand side management, charging, combinatorial algorithm, graph search algorithm, indirect control by variable tariffs

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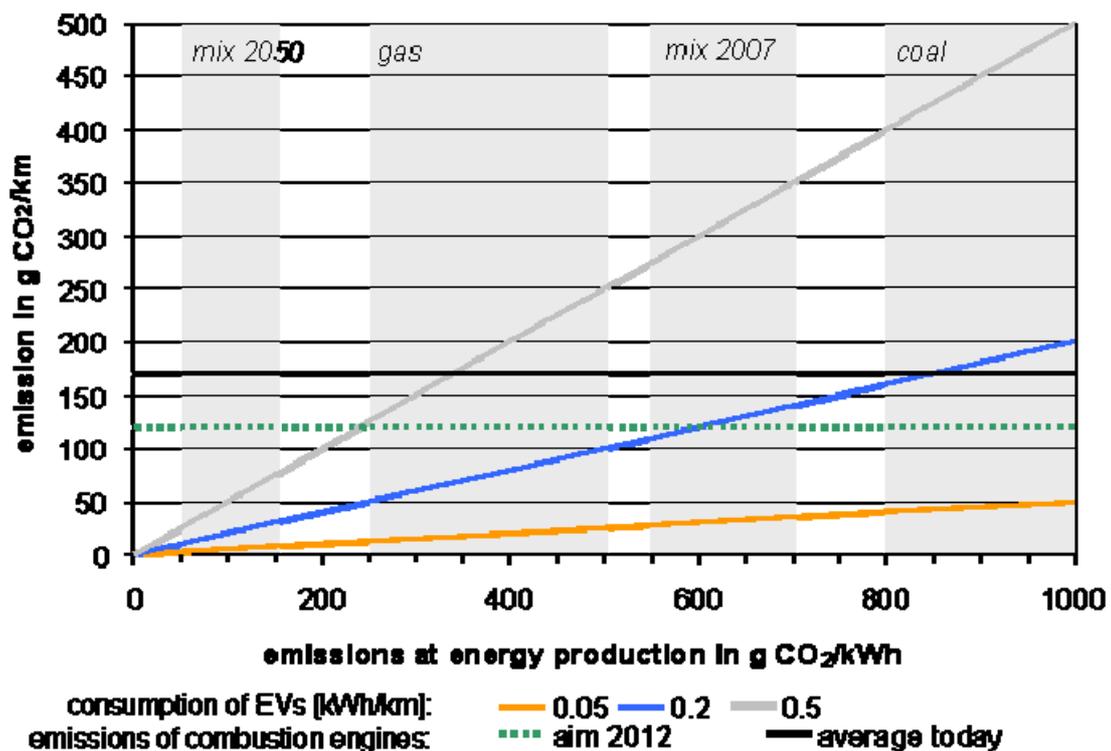
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## 1 Introduction

By successfully launching hybrid technology into the market and through additional progress in battery development, electric mobility is again considered capable of becoming a viable option for a marketable vehicle concept. Promising advantages of battery electric vehicles (BEV) are not only high efficiency, reduction of local and total emissions, but also a better integration of fluctuating renewable energy and the possibility of highly efficient grid operation by intelligent integration of BEVs, too.

Figure 1: BEV CO<sub>2</sub> emissions per km as a function of the used energy and their consumption. The grey highlighted areas illustrate different sources assumed for the electrical energy production. Mix 2050 is the energy mix in 2050 used in Germany in 2050, mix 2007 stands for a production mix according to a pilot study of the federal ministry of environment (BMU) for Germany



In order to illustrate the connection between CO<sub>2</sub> emissions and sources of electricity generation, figure 1 compares emissions of different BEV pursuant to their consumption as well as average and targeted emissions for new vehicles with combustion engines. The emissions for produced electricity in gram CO<sub>2</sub>

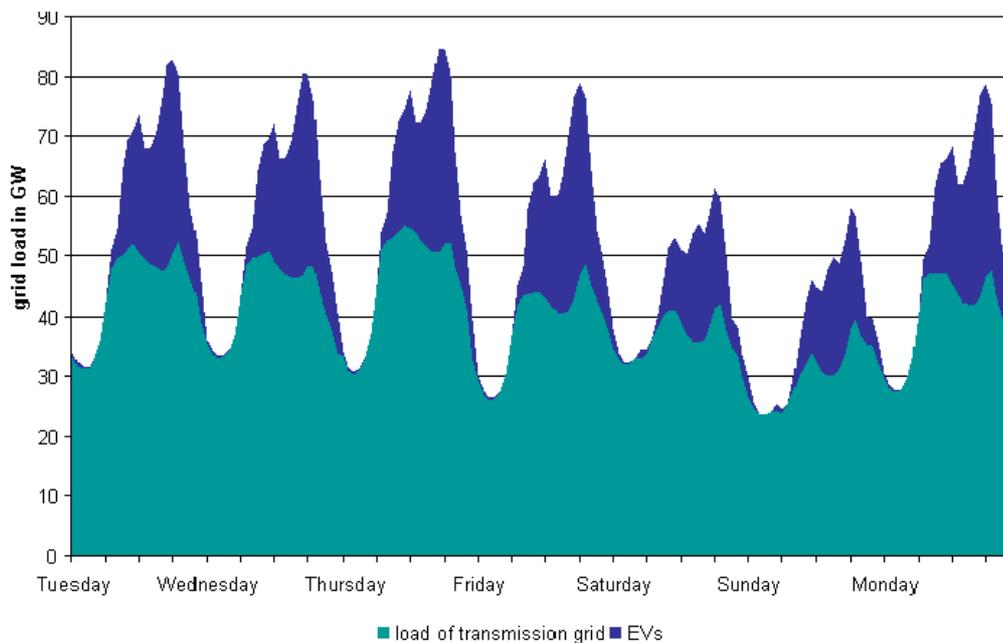
per kWh are plotted on the x-axis. The grey highlighted areas point out ranges for emissions based on different sources of electricity generation or mixes of sources. For example, the CO<sub>2</sub> emissions of coal power plants vary from 800 gCO<sub>2</sub>/kWh to more than 1,000 gCO<sub>2</sub>/kWh depending on power plant efficiency as well as quality of coal. Mix 2007 highlights the CO<sub>2</sub> emissions of the German energy mix in 2007. Mix 2050 illustrates the CO<sub>2</sub> emissions if the German goals for 2050 according to the pilot study from [1] will be reached. The emissions in gram CO<sub>2</sub> per km, plotted on the y-axis, are due to the coherence between emissions per kWh and the consumption kWh/km. The black line shows average CO<sub>2</sub> emissions of today's cars with combustion engines in Germany (~170 g CO<sub>2</sub>/km). The dotted green line indicates targeted emissions for vehicles with combustion engines in 2012 (130 gCO<sub>2</sub>/km). The diagonal grey, blue, and orange lines stand for the different exemplary energy consumptions of BEV so that the lowest line (orange) e.g. demonstrates the consumption of a CityEL (0.05 kWh/km [2]). To reduce emissions with BEVs the supplying fuel source for electrical driving is crucial. Only by supplying e-mobility with renewable energy is a significant CO<sub>2</sub> reduction within reach.

The expected immense expansion of renewable and decentralised energy systems [1] will generate a high supply of fluctuating energy in the grid. By focusing on the German situation, the German Association for Electrical, Electronic & Information Technologies VDE is convinced that the necessary expansion of the grids can be delayed or even avoided by using controllable decentralized power plants and/or storage systems [3]. Energy systems with a high share of fluctuating generation need demand side management to meet the supply curve. Therefore it is necessary to implement adjustable load and/or storage systems. From 2011 onwards, German customers of household electric connections have the right to claim for demand or time variable tariffs [4]. These tariffs should provide an incentive to save energy and shift the energy demand to the time of production. The handling of these tariffs can be easily implemented with smart electric meters since they are able to show actual demand and time of use. At the beginning 2010 it becomes mandatory to install smart electric meters in new buildings and during renovations. These meters will lead to new possibilities for the billing of BEVs. If customized tariffs for households are realised, special tariffs for the new load from BEVs are a possible next step.

The required power is the key parameter for BEVs, not the required energy. At a penetration of 10% of German passenger cars, the energy demand would be approximately 11.8 TWh (0.2 kWh/km as average BEV consumption and

around 41.3 million passenger cars in Germany) which is approximately 2% of the German gross energy production and approximately 27% of the produced solar and wind electricity in Germany in 2008 [5]. In figure 2 a weekly load profile of the German transmission grid from October 2008 and the simulated additional load by 100% substitution of passenger cars with BEVs charged directly after each trip is shown exemplarily. The graph illustrates just as [5] [6] [7] and other publications have also pointed out, that BEVs create demand peaks at similar time frames like the peaks in the existing electric energy profile. To avoid grid stress it is necessary to control and shift the charging process of BEVs. Parking times of passenger cars are on average between 93 and 96% of their lifetime [8] and so shifts of the charging times are often possible without negative impacts. The possible load shifting times and the indirect control of grid-connected battery vehicles are shown in [9]. BEVs could also be used to provide auxiliary services. As auxiliary services in this context are meant feeding electricity back into the grid, stopping and starting the charging process. In [10] Williett Kempton established the wording “Vehicle to grid” (V2G) to describe the electric power resources from vehicles.

Figure 2: Load of the German energy transmission grid with the simulated energy demand for BEV penetration of 100% passenger cars. The graph underlines the fact that the timeframe of energy demand and demand peaks of BEVs overlap



**Assumptions:** 3.5 kWh grid connection; 90% efficiency battery charging; electric vehicles (approx. 41.3 Mio.), thereof: BEV: 150 km electr. range and 0.18 kWh energy use per km

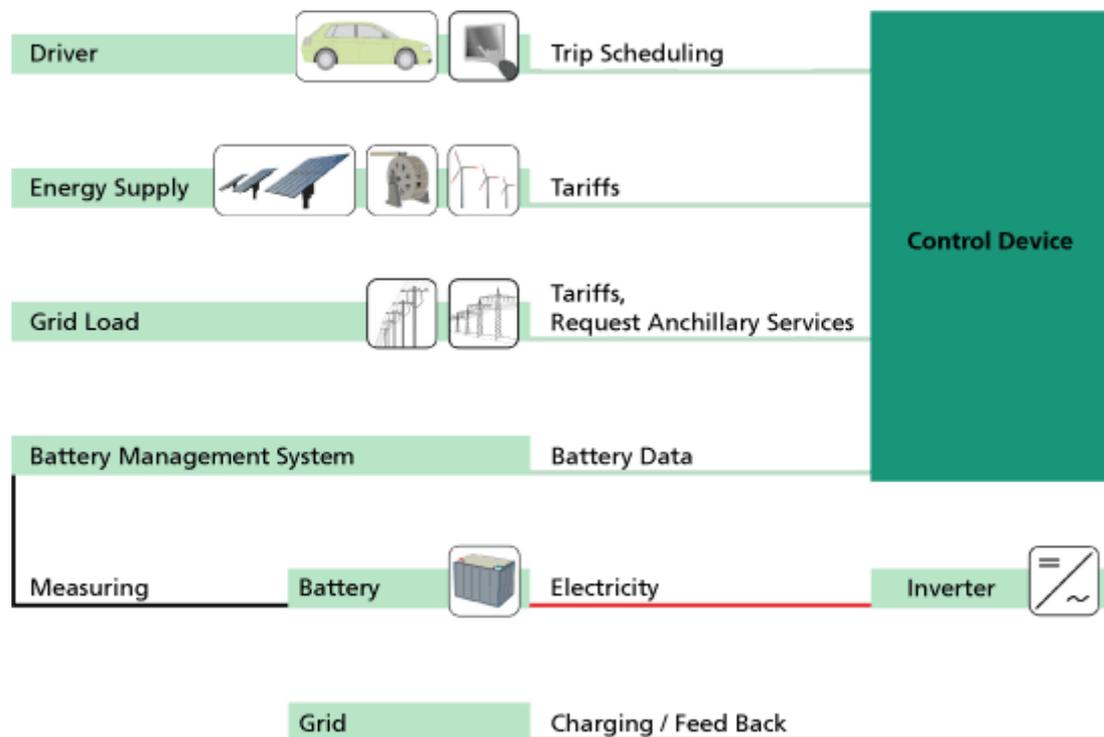
In [5] [6] and [11] coordinated and uncoordinated charging options for BEV are discussed. [6] also discusses differences in directly controlled and indirectly controlled charging processes. For coordinated directly controlled charging, a utility (for example the Distribution System Operator (DSO)) plans and controls the charging process for all cars within the system. For both, a directly and indirectly controlled charging communication infrastructure has to be installed. In [6] these infrastructural requirements are discussed and have been proved in and for practise in many fleet tests and projects all over the world.

Indirect control via variable electricity tariffs and a mobile decentralised load dispatch device could be one possible solution to manage charging times. Among others, the variable tariffs indicate the electricity supply and demand. Thus, it is possible to charge the vehicle batteries simultaneously with electricity generation from renewable energies as requested for effective CO<sub>2</sub> reductions and the avoidance of grid stress. The suggested method to schedule the BEV charging process is to integrate an embedded control device into the vehicles. The control devices manage charging and discharging locally, according to variable tariff signals. In order to calculate the optimal charging schedule, the needs of the driver, utilities and vehicle specific data are taken into account which leads to optimisation tasks.

## 2 Different optimisation approaches

The cost-optimised charging of the BEV battery is the target function of the optimisation in the mobile dispatching device. Constrains are illustrated in Figure 3. The technical restrictions are: the charging infrastructure in use and the state and characteristics of the battery system. Economic terms depend on the contract for energy / power supply and provision. Further restrictions are driver demands (time and range for the next trip). Variable tariffs represent the energy supply and the energy demand. By shifting the charging time (or even a temporal discharge of the battery), energy cost could be reduced. The load shift also leads to a reduction of peak load in the grid. As a result of the optimisation, a charging and discharging schedule for the parking time should be calculated. This schedule will be used to control the charging of the BEV battery.

Figure 3: Outlines influencing factors for the charging optimisation of BEV



An optimised system will typically only be optimal in one application or for one audience. Different algorithmic approaches could be the choice for different requirements. To implement the mobile decentralised tariff-based charging schedule in an embedded computing system, a time, processing and resource effective algorithm is needed.

A general way to solve optimisation problems is by linear programming. Thereby, the problem is described with a set of variables. A special solver maximizes a linear target function under the restrictions of linear boundary conditions. If some elements of the solution have to be discrete (e.g. the state of a battery which is either “load” or “idle”), the problem may become very complex. In this case (mixed) integer linear programming is used. The runtime of such a solver can be exponential in terms of the input. The computation times of this approach solved by a commercial solver turned out to be too long for calculations on resource-efficient-embedded systems or grid impact calculations for a mass of BEV.

A different way to solve optimisation tasks is the use of combinatorial algorithms which provide simple rules on how a schedule should be created. The challenge of those algorithms is that the optimality and correctness of the calculated solution may not be definite. This work presents a combinatorial algorithm to calculate the optimised charging schedule for a BEV battery and proves its correctness and optimality. This algorithm uses a rather simple battery model without considering degradation costs. It is also shown how additional requirements can be integrated.

Important for the V2G option of feeding energy back to the grid are the battery degradation costs. Therefore highly simplified battery cost degradation models are used. These cost models are described later. One of these is a monotone decreasing function based on the depth of discharge (DoD). For this case, the developed combinatorial algorithm with all its strengths cannot be used and therefore a graph search algorithm is used. This algorithm will also be introduced and evaluated within this paper.

Discrete optimisation algorithms can solve optimisation tasks by graph search. If assumed that the battery only charges and discharges exactly 1 unit of energy, the system can only be in  $\mathbf{M+1}$  different states at every time step ( $\mathbf{M}$  is the max. capacity of the battery). If there are  $t_{\max}$  time steps, a graph of  $(\mathbf{M+1}) \cdot (t_{\max})$  nodes can be created, where each node represents a state and a time. Then edges are introduced which represent possible state transitions. Each edge can be assigned certain costs/gain which occur through this transition (e.g. costs for charging, gain for discharging). Finding an optimal plan can then be solved by finding the best path from an initial node at time 0 to a node at time  $t_{\max}$ . This method is also used for similar optimisation problems of energy systems with different components e.g. combined heat and power (CHP) plants storage and heating system.

### 3 The combinatorial algorithm

For the following algorithm it is assumed that the battery is empty at the start and end of the schedule. The battery has a maximum capacity  $M$  and can be charged and discharged at different, time-dependent prices. Additional requirements can easily be brought in later, but they make the algorithm and especially the following declarations and proofs more complex.

#### 3.1 Input values

The scheduling problem is formally defined as follows:

- $M$  is the maximum capacity of the battery.
- $tmax$  is the number of time steps for which a schedule must be generated and  $T = \{0, \dots, tmax\}$  is the corresponding set of time steps.
- $price_c: T \rightarrow R_0^+$  represents the costs for charging one unit of energy at time  $t$
- $price_f: T \rightarrow R_0^+$  represents the gain for feeding one unit of energy into the grid at time  $t$

An important precondition is that the gain for feeding energy into the grid is never higher than the costs for charging the battery at the same time:

**Precondition:**

$$\forall t \in T: price_c(t) \geq price_f(t)$$

This constraint avoids that the battery is charged and discharged at the same time.

#### 3.2 Result values

The result has to be a schedule  $P$ , which (1) satisfies the boundary condition and (2) maximizes the gain. It is defined as follows:

- $P: T \rightarrow \{-1;0;1\}$  is the schedule produced by the algorithm. For each time step  $t$  the function value  $P(t)$  could be:
  - $P(t) = -1$  means, that the battery feeds one unit of energy into the grid,
  - $P(t) = 1$  means, that the battery charges one unit of energy from the grid
  - $P(t) = 0$  means, that the battery is idle.

- **Gain(P)** =  $[\sum_{t \in T \wedge P(t) < 0} (-P(t) \text{ price}_F(t))] - [\sum_{t \in T \wedge P(t) > 0} P(t) \text{ price}_C(t)]$  is the gain of the schedule. It is the total gain for feeding into the grid minus the total cost for charging the battery.

The state of charge after a time step  $t \in T$  is  $C(t) = \sum_{i=0 \dots t} P(i)$ . We denote the initial state, before the first step of the plan, as  $C(-1)$ .

It is assumed that the battery is initially and finally empty, i.e.  $C(-1) = 0$  and  $C(t_{max}) = 0$ . The state of charge is never less than empty or more than  $M$ , so that the following boundary condition results:

**Boundary Condition:**

$$\forall s \in T: 0 \leq C(s) \leq M$$

### 3.3 Variables used by the algorithm

An important property of the algorithm is that it uses a sorted list of tuples  $L = \{(p_1, l_1), (p_2, l_2), \dots\}$ . Each tuple  $(p_i, l_i)$  represents a time step  $l_i \in T$  and a price  $p_i \in R^+$ .

The list has two properties:

- 1)  $L$  is sorted by the price  $p_i$  in ascending order. The first element has the lowest price  $p_1$
- 2)  $L$  contains at most  $M$  elements. If  $L$  temporarily contains more than  $M$  elements, those elements at the end of the list, i.e. with the highest price, have to be removed.

The algorithm starts with an empty list  $L$  and runs through all time steps  $t$  from  $0$  to  $t_{max}$ . Thereby the list stores information about time steps before  $t$ , at which units of energy can be obtained. I.e. if, at a time step  $t$ , a unit of energy can be charged from the grid at price  $\text{price}_C(t)$  a tuple  $(\text{price}_C(t), t)$  is inserted into  $L$ . If the algorithm later decides to use that unit of energy for later discharging, the tuple is removed from the list. Additionally, every discharging at time  $t$  for discharging price  $\text{price}_F(t)$  also adds a tuple  $(\text{price}_F(t), t)$  to  $L$  since the decision to discharge can be undone to get back that unit of energy for later use. As we see, there are these two possibilities of obtaining energy, both by charging and by undoing discharging decisions.

### 3.4 The algorithm

The algorithm creates a schedule  $P_{Alg}$  as follows. Further on the algorithm is explained, by referring to the line numbers on the left:

```

 $L := \{\}$ ;           // The list is initially empty

 $P_{Alg} := 0, \forall t \in T$ ;      // The schedule is initially idle for all time steps

1. FOR  $t := 0$  TO  $t_{max}$  DO      // go through all time steps
   {
2. IF ( $L \neq \{\}$ )
3.  AND IF ( $p_1 < price_F(t)$ ) THEN // ( $p_1, l_1$ ) is the first tuple of the list  $L$ 
   {
4.   $P_{Alg}(l_1) := P_{Alg}(l_1) + 1$ ;
5.   $P_{Alg}(t) := P_{Alg}(t) - 1$ ;
6.  REMOVE ( $l_1, p_1$ ) FROM  $L$ ;
7.  INSERT ( $t, price_F(t)$ ) INTO  $L$  (sorted);
   }
8.  INSERT ( $t, price_C(t)$ ) INTO  $L$  (sorted);
9.  IF ( $|L| > M$ ) THEN
10.   TRIM  $L$  to the first  $M$  elements;
   }

```

*In words:*

While going through all time steps (Line 1), the elements of the list  $L$  contain time steps before time step  $t$  at which a unit of energy can be obtained. The cheapest unit of energy which is available is the first tuple in the list  $(p_1, l_1)$ . If at

a time step  $t$  the gain for feeding into the grid is higher than the costs for obtaining the energy at an earlier point of time  $I_1$  (Line 3), then the algorithm obtains one unit of energy at time  $I_1$  (Line 4) and feeds it into the grid at time  $t$  (Line 5) and thereby increases the total profit of the plan.

The next step of the algorithm is to update the list  $L$ : after one unit of energy is obtained at  $I_1$  (Line 4), the algorithm has to remove this option of obtaining energy from the list (Line 6). Instead, one additional unit of energy can now be obtained at time step  $t$  and at the cost of  $\mathit{price}_F(t)$  by undoing the discharging and setting the schedule  $P_{Alg}(t)$  from  $-1$  to  $0$  again (Line 7).

In all cases, one unit of energy can later be obtained by charging at time step  $t$  at the cost of  $\mathit{price}_C(t)$ . Hence, the tuple  $(t, \mathit{price}_C(t))$  is inserted into the list  $L$  (Line 8). And at last, the amount of energy which can be charged before  $t$  and used later, is limited by the capacity of the battery. That is why  $L$  is trimmed to contain at most  $M$  elements (Line 10).

Note that a time step  $t$  might occur twice in the list (Lines 7 and 8), if  $P_{Alg}(t) = -1$ , because by changing  $P_{Alg}(t)$  to  $0$  and then to  $1$  two units of energy can be obtained. In that case, the price for getting the first unit of energy is  $\mathit{price}_F(t)$  and for getting the second unit of energy is  $\mathit{price}_C(t)$ . But by the precondition and the ordering of  $L$ , the tuple  $(t, \mathit{price}_F(t))$  is before  $(t, \mathit{price}_C(t))$  in the list, hence the first unit of energy obtained from time step  $t$  is correctly price  $\mathit{price}_F(t)$ .

### 3.5 Proof of correctness

The following paragraph shows that the algorithm satisfies the boundary condition after every loop (Line 1). By the definition of the state of charge we can write the boundary condition as:

$$\forall s \in T: 0 \leq C(s) \leq M$$

$$\forall s \in T: 0 \leq \sum_{i=0 \dots s} P(i) \leq M$$

1. First, it is shown that  $\forall s \in T: 0 \leq \sum_{i=0 \dots s} P(i)$ . <sup>(1)</sup>:

We show by induction that this condition is satisfied after every loop  $t$  of the algorithm (line 1), which implies that the condition is also satisfied for the resulting plan.

At the beginning, the algorithm sets  $P_{Alg} := 0, \forall i \in T$  and hence the sum  $\sum_{i=0..s} P(i)$  is also  $0$  for all  $s \in T$  and condition (1) is satisfied.

Now assume that (1) is satisfied at the beginning of loop  $t \in T$ .

If the IF-condition in line 3 is not satisfied,  $P_{Alg}$  does not change within this loop, and the condition (1) is still satisfied at the beginning of the next loop  $t+1$ .

If the IF-condition in line 3 is satisfied,  $P_{Alg}(I_1)$  increases by  $1$  and  $P_{Alg}(t)$  decreases by  $1$  where  $I_1 < t$ .

Thus the sum  $\sum_{j=0..s} P_{Alg}(j)$  increases by  $1$  for  $I_1 \leq i < s$  and stays constant otherwise. Hence, condition (1) is still satisfied at the beginning of loop  $t+1$ . This means that condition (1) is still satisfied at the end of the algorithm.

2. Next is shown that (2)  $\forall s \in T: \sum_{j=0..s} P_{Alg}(j) \leq M$ :

Be  $s \in T$ : at the beginning,  $P_{Alg} = 0, \forall i \in T$  and hence the sum  $\sum_{i=0..s} P(i)$  is also  $0$  and condition (2) is satisfied.

This sum can only change at the lines 4 and 5, since only those lines change  $P_{Alg}$ .

$$4. P_{Alg}(I_1) := P_{Alg}(I_1) + 1;$$

$$5. P_{Alg}(t) := P_{Alg}(t) - 1;$$

These statements only increase the sum (2)  $\sum_{i=0..s} P_{Alg}(i)$  by  $1$ ,

if  $I_1 \leq s < t$ . This only happens at a loop  $t > s$ , and at most once per tuple in the

List  $L$  at time step  $s$ , since in that case the tuple  $(I_1, \dots)$  must have been inserted into the list at loop  $I_1 \leq s$  and remained in that list at the end of loop  $s$ . Since there are at most  $M$  elements in the List  $L$  at loop  $s$ , the sum (2) can increase by  $1$  at most  $M$  times, hence  $\sum_{j=0..s} P_{Alg}(j) \leq M$  is still satisfied when the algorithm ends.

From 1 and 2 it follows that the boundary condition is satisfied by the schedule  $P_{Alg}$ .

### 3.6 Proof of optimality

Every discharged unit of energy has to be charged at an earlier time step. Hence an optimal schedule **Opt** can be constructed by going through all time steps and determining an earlier charging time for each time step. As in our algorithm, we can assume that also for the construction of the optimal plan **Opt**, the best **M** options for obtaining energy are stored in a list **L**.

Now we can easily show that the construction of an optimal plan can be done in exactly the same way as by the combinatorial algorithm.

When a unit of energy is charged by the optimal plan, we can w.l.o.g. also assume that it is the cheapest option to obtain energy in the list **L**. Otherwise, if this unit of energy would not be used by the optimal plan, the plan could be improved by using the cheaper option. This would contradict the optimality.

As the combinatorial algorithm, the optimal plan will also only charge and discharge a unit of energy, if the price for energy is not higher than the gain for discharging. Else, the plan could be improved by simply leaving out this pair of charging and discharging, which contradicts the assumption of optimality.

On the other hand, when **Alg** plans to charge and to discharge energy (and thereby making profit), while the optimal plan does not, this must imply that the optimal plan charges the same unit of energy too, but discharges this unit of energy later (otherwise, the optimal plan could be improved by adding this pair of obtaining and discharging energy – which would contradict the optimality). In that case, we could w.l.o.g. assume that the optimal algorithm also plans to obtain and discharge that unit of energy as the combinatorial algorithm does, and later obtains back that unit of energy by removing the discharging from the plan again.

As we see, we can assume that an optimal plan is constructed in the same way as the plan by our combinatorial algorithm. Hence the combinatorial algorithm is optimal.

### 3.7 Constraints and possible add-ons of the combinatorial algorithm

As said before, the algorithm described in detail is a simplified version, add-ons for practical usage are:

- An initial state **i** of the battery: it can be included by adding additional time steps at the beginning of the time line. By adding **s** additional **i** time steps

with  $\mathbf{price}_c(t)=0$ ,  $\mathbf{price}_f(t)=0$  at the beginning, a schedule can use  $\mathbf{s}$  free units of energy for the schedule.

- A final state  $\mathbf{f}$  of the battery: it can be included by adding time steps at the end of the time line. By adding  $\mathbf{f}$  time steps with  $\mathbf{price}_c(t)=X$ ,  $\mathbf{price}_f(t)=X$  ( $X$  sufficiently large) at the end of a schedule, an optimal schedule will (if possible) charge the battery to  $\mathbf{f}$  before these additional time steps.
- Different charging and feeding rates: basically, the list  $\mathbf{L}$  must be modified to contain triples  $(l_i, p_i, e_i)$ , where  $e_i$  is now the amount of energy which can be obtained at time step  $l_i$  at price  $p_i$ . Instead of having  $\mathbf{M}$  elements, the list must now be truncated to contain at most  $\mathbf{M}$  units of energy. We therefore do not just remove triples from the list, but also reduce the energy contained in the last triple. As before, the algorithm obtains the energy from the triples at the beginning of the list  $\mathbf{L}$ , but now this energy may be divided over several tuples. So again, several triples may be removed from the list, or the energy contained in those triples may be reduced.

At this stage only a very simplified model for the battery is included in the described algorithm, possible function plug-ins are:

- Monotonic rising battery state of charge depending on abrasion costs for each time step. (e.g.: if the battery contains 5kWh, abrasion cost of  $XX$  € for each time step incurred)
- Losses of the battery: the battery loss could depend on the batteries SoC. Also possible are losses for each time step, depending on the SoC. For example,  $X\%$  losses per time step at SoC greater  $Y\%$  and  $Z\%$  losses at  $Y\%$  and less.
- Monotonically increasing costs for charging depending on charging power: this can be achieved by inserting several small units of energy with different charging prices into the list  $\mathbf{L}$  instead of one.

### 3.8 Complexity and time consumption

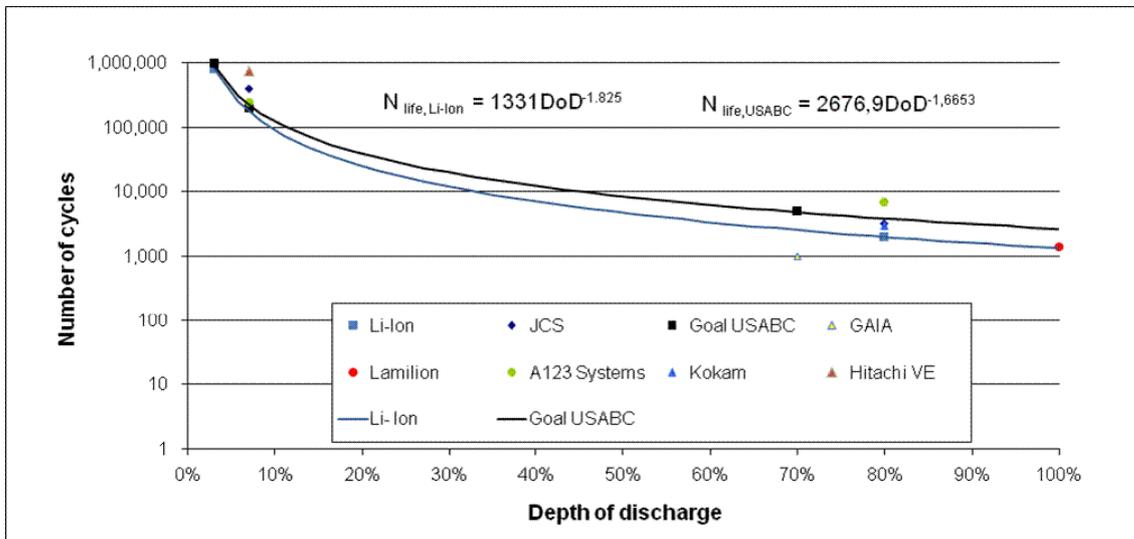
The complexity of the combinatorial algorithm in the simple version is linear to the required time steps list. In practice, on a 2.5 GHz ATLON 64 4000+ processor this algorithm consumed just  $7 * 10^{-5}$ s computation time for a 36-hour schedule based on quarter hourly time steps. The fast computation time allows large simulation runs. For example, the algorithm has been successfully applied to evaluate the impacts of a high penetration of Plug-In Vehicles indirectly controlled via variable tariffs on the grid. Also fast calculations of onboard schedules for charging strategies are possible.

## 4 Used battery degradation cost model

An important prerequisite for the V2G use of batteries depending on variable tariffs is the battery degradation. Only when the spread between high and low tariffs is large enough to cover the battery degradation costs plus buying electricity feeding-back is suggestive. The battery degradation process is too complex and theoretically not well enough understood to use a physical battery degradation model in each solver. Therefore, a highly simplified battery degradation cost model based on the depth of discharge (DoD) was constructed and will be used in the fleet test “Flottenversuch Elektromobilität” (see also further on). This model is used as a first assumption, as explained before; different approaches could also be integrated into the optimisation.

To illustrate the typical battery degradation, Figure 4 shows the cycle life of different battery cells published in [12] and the goal of the U.S. Advanced Battery Consortium (USABC) published in [13]<sup>1</sup>.

Figure 4: Battery cycle life dependent on DoD and battery degradation trend line



Own calculations based on data in [12] and [13]. For shallow cycles a 7% DoD is used. The battery ageing due to calendar life is not taken into account.

To estimate the cycle life performance of a currently available battery, a lifetime<sup>2</sup> of 2,000 cycles at 80% DoD and 800,000 cycles at 3% DoD is used. This

<sup>1</sup> For the USABC trend line, a cycle life of 1,000,000 at DoD of 3% was suggested.

performance has been demonstrated for a Li-Ion high energy cell of SAFT and seems to be feasible for Li-Ion batteries today (see [14]).

The following equation is used to estimate cycle life  $N_{life}$  in dependence of the  $DoD$ .

$$N_{life} = a * DoD^b$$

A DoD of 100% is defined as the usable energy of the battery. For a currently available Li-Ion battery, parameters ( $a=1331$  and  $b=-1,825$ ) are used. Representative for the USABC goal, the parameters ( $a=2744,2$  and  $b=-1,682$ ) describe the cycle life with 5,000 cycles at 80% DoD and 1,000,000 cycles at 3% DoD. The equation is used to calculate the cycle life in the usable energy range of Li-Ion batteries<sup>3</sup>. Nevertheless, a degree of uncertainty remains in this highly simplified model, since important parameters such as temperature, C- rate, specific aspects of different Li-Ion battery chemistries, or battery dimensions were not taken into account. Moreover, the battery ageing due to calendar life is unaccounted. The discussed model suggests the highest life time for a fully charged (100% SoC) battery without cycling. However, considering calendar life, a SoC of 100% is the most demanding condition. Furthermore, especially for A123 Systems batteries, the dependency of cycle life and DoD seems not to be the appropriate approach. Analyses of [15] show that the most important factor for capacity fade of A123 Systems is the energy processed, and not the DoD which is used in the equations. According to the A123 Systems website, a cycle life of 7,000 cycles for a capacity fade of 20% is assumed. This results in a lifetime reduction of 0.0029% per cycle<sup>4</sup>. Peterson concludes that capacity fade per normalized Wh processed is 0.0062% for driving and 0.0027% for arbitrage. The disparity of the two values is caused by different C-rates of the driving and arbitrage cycling.

#### 4.1 Discharge costs

To decide if V2G options are profitable, the battery degradation costs per unit discharge are required. When the battery is discharged the degradation costs are a function  $c_{dis}(DoD_{Start}, DoD_{End})$ , which depends on the  $DoD$  at the start of

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2 80% of original capacity.

3 Different types and specific aspects such as safety of Li-Ion batteries are not considered.

4 1C/-1C cycling (25°C, 3.6 V, DoD 100%).

the discharging ( $DoD_{Start}$ ), the  $DoD$  at the end ( $DoD_{End}$ ). Additional parameters of the function are battery-specific parameters, the cost for the battery  $c_{Batt}$  and the usable energy of the battery  $E_{Batt}$ . We now consider the special case of regular charging and discharging up to a certain  $DoD$ , assuming that the degradation costs are equally distributed over all life cycles of the battery. In that case the costs for one cycle, i.e. one discharge from  $DoD_{Start}= 0$  to  $DoD_{End} = DoD$  are the total battery costs divided by the number of cycles:

$$c_{dis}(0, DoD) = c_{Batt} / N_{life}(DoD)$$

The costs for one kWh proceeded illustrated in Figure 5 are:

$$c_{dis, energy}(0, DoD) = c_{Batt} / (N_{life}(DoD) * DoD * E_{Batt})$$

It follows that under the given assumptions the general degradation costs are:

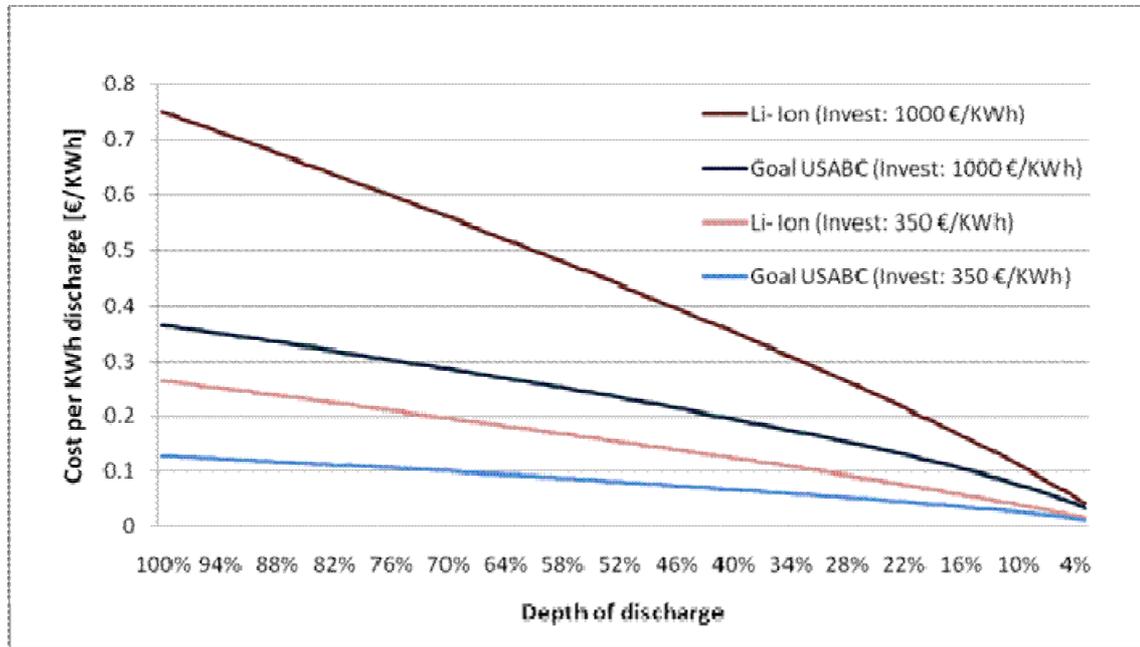
$$c_{dis}(DoD_{Start}, DoD_{End}) = c_{dis}(0, DoD_{End}) - c_{dis}(0, DoD_{Start}) \text{ for } DoD_{End} > DoD_{Start}$$

Then, the cost per discharge unit  $c_{dis\_unit}$  as a function of the  $DoD$  before the discharge are:

$$\begin{aligned} c_{dis\_unit}(DoD) &= c_{dis}(DoD, DoD+1) \\ &= c_{dis}(0, DoD+1) - c_{dis}(0, DoD) \\ &= c_{Batt} / N_{life}(DoD+1) - c_{Batt} / N_{life}(DoD) \end{aligned}$$

Figure 5 illustrates these specified discharge costs as a function of the  $DoD$  for both earlier described degradation functions with specific investment costs of 1,000 and 350 € per kWh of usable energy.

Figure 5: Battery degradation costs for a currently available battery (Li-Ion) and a battery with cycle life given by the USABC goal



For the two cycle life functions, a deep discharge and an investment of 1,000 €/kWh results in very high discharge costs per kWh. In the USABC scenario with low battery costs (350 €/kWh), costs of discharging range between ~0.04 and 0.13 €/kWh. The average spread between base and peak prices at the European Energy Exchange (EEX) market in 2008 was in the range of 0.03 €/kWh. This relation shows that feeding electricity back into the grid only occurs in very limited time frames and in a best case scenario.

Bringing the focus back again to the A123 Systems cells<sup>5</sup> and assuming an investment of 1,000 and 350 €/kWh, respectively, for the total battery system, the costs per kWh discharged would result in 0.143 and 0.05 € respectively. Especially in the case of California which reveals a higher spread between peak and base electricity prices, and for energy systems with a very high penetration of volatile generation, the A 123 Systems is very promising for the future. Nevertheless, it remains unclear if the assumed cost reductions can be reached. In order to analyze possible effects of V2G, in the goals of the USABC are used in this paper as the most reasonable future scenario. The required infrastructural

<sup>5</sup> We assume 7,000 cycles for a capacity fade of 20% for the battery system. However, cycle life of cells and the systems constructed of several cells can differ strongly.

implementation costs for charging equipment and metering also have to be considered.

Integration of the battery degradation in the charge optimisation is only required for the bidirectional usage of the batteries (V2G integration). In the combinatorial optimisation described earlier, only monotonic rising battery state of charge depending on abrasion costs for each time step could be included. In the mentioned battery function, however the charge depending on abrasion costs is decreasing. Hence, a graph search algorithm is used for all V2G options.

## 5 Graph search algorithm

In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) such that the sum of the weights of its constituent edges is minimized. From a given initial vertex (node), the graph search algorithm calculates the path with lowest cost between that vertex and every other vertex in the graph. For the charge optimisation issue the battery state of charge (SoC) at arrival could be used as initial vertex. From this initial vertex to every other reachable vertex (SoC at the next time step) in the system, a cost value is assigned. Constraints of the passages between different vertices depend on the SoC and other parameters like charging and discharging energy. The cost for the passages between the different vertices depends on the battery degradation costs, a function of SoC, as shown and the difference between time-specific energy costs and feed-in tariffs. For all possible battery states at the specific time, vertices were defined. Also for all passages between the vertices the costs were defined. The framework of the specific vertices and the cost-specified passages between them are called graph. Starting from the initial vertex, different end vertices are possible. In the example also the end vertex is more or less fixed. Minimum should be the SoC to reach the desired destination of the next trip. With a shortest path algorithm akin to Dijkstra's [16] algorithm, the cheapest passage through the graph could be calculated and so the schedule with the best charging/discharging strategy is found.

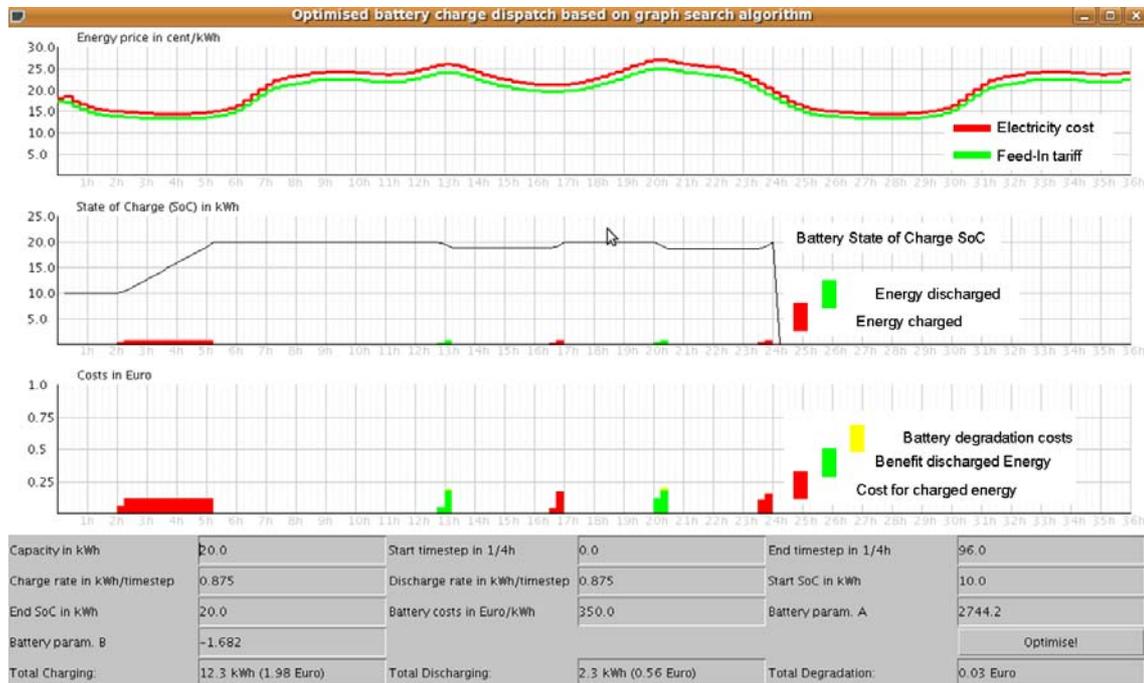
### 5.1 Complexity and time consumption

The complexity of the graph search algorithm is linear to the number of vertices. Therefore, discrete variables which allow the required level of detail, without the expansion of the possible vertices, are needed. In practice, on a 2.5 GHz ATLON 64 4000+ processor this algorithm consumed around 1s computation time for a 36-hour schedule, based on quarter hourly time steps and up to 40,000 discrete states of the battery. To schedule onboard the charging and discharging strategy incl. a model of the battery degradation costs, the graph search algorithm is a good choice. For large simulation runs e.g. to evaluate the impacts of variable tariffs on the grid, the combinatorial algorithm should be the favoured choice.

## 6 Visualisation and applications

Both algorithms are designed for resource effective embedded systems and therefore without visualisation. But for the developing process and also for an easier explanation and evaluation of the functions of both algorithms, an input interface visualisation is used. Figure 6 shows a screen shot of the Java visualisation for an exemplary schedule. The diagram above shows the assumed variable electricity cost and feed tariff, in the example both curves are related to the German reference load profile (SPL H0<sub>Winter-Workday</sub>). Further assumptions in the example are the shaded input parameters at the bottom of the visualisation. In the example, the car arrives with a SoC of 10 kWh and 24 hours later it has to be fully charged (20 kWh). A one phase grid connection point common in Germany (max. ~3.5 kW) and the cost and cycle life of Li-Ion batteries given by the USABC goals are used. The optimised dispatch schedule is illustrated in the other diagrams of the Java visualisation applet. The black line in the diagram which is shown in the middle of the graph shows the SoC. The green and red bars illustrate the amount of energy charged and discharged for each time step. At times when the tariffs for drawn energy are low (in the morning between 2 and 5 o'clock), the battery is charged fully. Also at times when the energy prices are below the peak of the feed-in tariff, the battery is charged to feed energy back at the times of high feed-in tariffs, around noon or around 8 pm. Energy is only fed back if the spread between electricity costs and feed-in tariffs is higher than the battery degradation costs. In the third diagram the energy cost and benefit for each time step are illustrated. The total amount of the green and yellow bars stands for the money which could be earned by feeding energy back. The yellow part illustrates the battery degradation cost for which the calculation was explained before. The green bar illustrates the profit and the red bar the cost for the consumed energy.

Figure 6: Visualisation and graphical interface of the optimised battery charge dispatch based on the graph search algorithm



## 6.1 Related project and research

Within the project “electro-mobility fleet test” (Flottenversuch “Elektromobilität”) both algorithms are used. Besides the partners Volkswagen AG and E.ON Energie AG the institutes Fraunhofer ISE and ISI are part of the project which is co-financed by the German Federal Ministry for the Environment, Nature Conservation and Nuclear Safety BMU [17]. The main task of the fleet test is to evaluate the interrelation between volatile renewable energies and the location-independent demand of plug-in cars. Unlike most fleet tests in the past, this project includes grid fitted, controlled charging and discharging. Therefore an intelligent operation management has to be implemented that generates an optimised charge and discharge strategy derived from different factors (grid operator, infrastructure, user needs, etc.). Also the bi-directional connection of the plug-in cars to the energy supply is planned. This implies new prospects by the mobile storage within the grid, new requirements of the cars, infrastructure, and communication. Currently, no infrastructure to independently bill mobile electrical loads exists. The prospective adaptation of “Smart Metering Systems” could be an opportunity to bill mobile electric loads. In the fleet test an onboard device to:

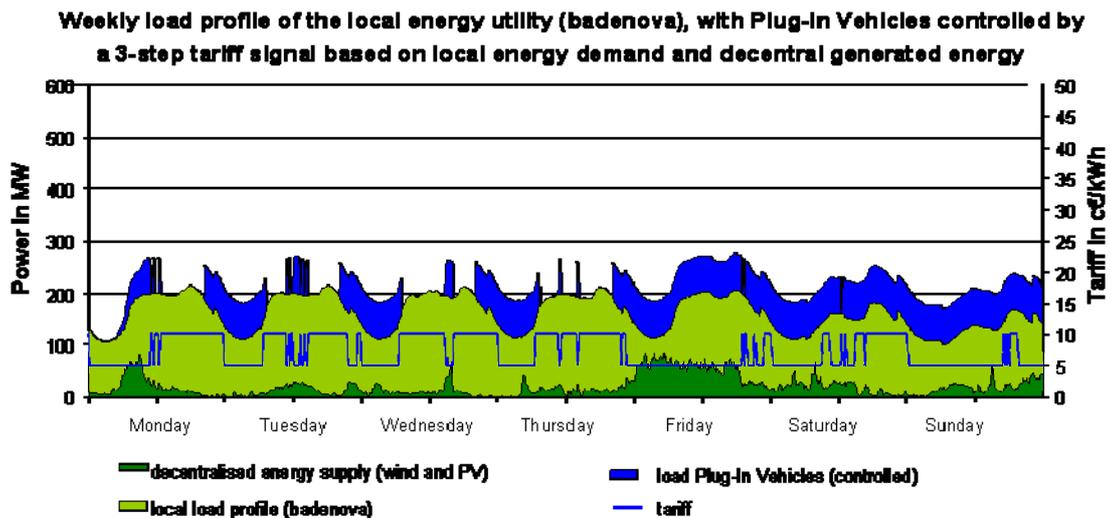
- communicate bi-directionally with charging station and utility (for authentication, to transfer the tariffs and load profile, etc.)
- meter the charged and discharged energy
- dispatch the energy consumption and optimised V2G applications
- store required data

is going to be developed. Depending on the requirements of the cars, the combinatorial (only charging) or the graph search algorithm (V2G) is used on the embedded computing system of the onboard metering device.

Both introduced algorithms are developed under the assumption that the final charging dispatch strategy is decided decentralised on board the BEV, so the required grid interface (charging station) could be relatively simple (see also [6], [18] and [19]). In contrast to the introduced concept and fleet test recently started, fleet tests in Germany [20] [21] do not realise direct communication between charging stations and the cars. If the charging schedule is decided off board, as is the solution within these fleet tests, the algorithms could be used to predict the reaction and energy demand, based on centrally generated signals. For both optimisation algorithms, add-ons to consider power and specific costs for energy produced and consumed locally (e.g. PV, CHP, etc.) already exist. This application could be used for “smart home charging” processes, considering the local energy situation within the charging schedule.

A further task done by the combinatorial algorithm is to evaluate the grid impacts of directly and indirectly controlled charging processes. (e.g. see also [5], [6]) Tariffs variable in time, power and location could be easily evaluated with the tool. It could also be used to develop these tariffs. In Figure 7 the impacts of a high marked penetration of Plug-In vehicles and charging indirectly controlled by tariffs is illustrated for the local energy utility. The two-step tariff signal links the local renewable energy production from wind and solar power with the load of the distribution grid in Freiburg. After evaluating the impacts of controlled and uncontrolled charging for different types of distribution grids, the impact for transportation and the link to the different fossil and renewable energy producers and decentralised energy producers and consumers will be evaluated. The resource-efficient combinatorial algorithm is adequate for all researches with a finite calculation capacity like embedded systems in mobile applications or mass calculations like reaction of many different Plug-In vehicles and their grid impacts.

Figure 7. Simulation of a local distribution grid load profile with a high penetration of BEV. The BEV Load is based on a three-step tariff signal and an onboard charging optimisation by the combinatorial algorithm. The exemplarily used tariff signal is only theoretical to describe the link by the tariff with the electrical load and the renewable energy production



Besides applications for BEV, simple modified versions of both algorithms could be and are in use to calculate optimised control schedules for decentralised energy systems, like virtual power plants. In consideration of the required developments to integrate distributed generators into distribution grids, the German legislator enacted amendments of the German Renewable Energy Sources Act (EEG) and Combined Heat and Power Act (KWKG) to improve the efficiency of the grid by creating incentives for local consumption of decentrally produced energy. This leads to the challenge of optimising local energy systems in a manner to cover as much as possible of local electricity demand by own plants. For solving these tasks, prognosis and optimisation algorithms are necessary and done by algorithms based on the described. For Plug-In Vehicles these amendments of both German laws are a first step to profit from a locally optimised charging dispatch depending on local energy production.

## 7 Conclusion

By successfully launching hybrid technology into the market and through additional progress in battery development, electric mobility is considered capable of being a viable option for a marketable vehicle concept again. Effective grid operations can be assured by synergy effects between the fluctuating energy supply of renewable energies and flexible load and storage capacity of electric vehicles. To be able to use the advantages of BEVs and realize large-scale operation of BEVs, charging and discharging has to be controlled and new infrastructures for communication of the grid and the billing system are necessary. An onboard “metering and control device” and variable energy/feed-in tariffs are an optional solution to managing charging and discharging. Effective optimisation algorithms are required to provide the optimal charging and discharging strategies of the battery system and to analyse the grid impacts. Therefore a combinatorial optimisation algorithm was developed, explained, and proved. In future the BEVs could be used to serve as energy storage, feeding electricity back into the grid when needed (V2G). In this case, battery degradation costs have to be considered. Based on the battery state of charge (SoC), common Li-Ion battery life cycle assumptions have been used to identify the degradation costs. To implement these battery degradation costs, an additional algorithm based on a graph search algorithm was used and customised. In the paper both algorithms were explained and compared by performance. On a common PC for a 36-h charging schedule the computation time of the algorithm based on graph search time is around a second which is around 50 times faster than solved by a commercial solver based on mixed integer linear programming (MILP). For the same task solved by the developed combinatorial algorithm the computation times last around  $10^{-5}$  seconds. Research projects, a visualisation and applications where the different algorithms are used and could be used have been shown. Both algorithms could be used to optimise the costs of the charging process onboard. Therefore, the influence on the grid of e-mobility is controllable by providing sufficient, dynamic tariff signals. This can lead to the necessary decoupling of driving and charging profiles in order to supply the electric passenger cars with the highest share of sustainable energy as possible without generating the necessity of expensive grid expansion or jeopardise the safe operation.

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